SENIOR ‘KANGAROO’ MATHEMATICAL CHALLENGE

Friday 1st December 2017

Organised by the United Kingdom Mathematics Trust

The Senior Kangaroo paper allows students in the UK to test themselves on questions set for the best school-aged mathematicians from across Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. **Use B or HB pencil only** to complete your personal details and record your answers on the machine-readable Answer Sheet provided. **All answers are written using three digits, from 000 to 999.** For example, if you think the answer to a question is 42, write 042 at the top of the answer grid and then code your answer by putting solid black pencil lines through the 0, the 4 and the 2 beneath.

   Please note that the machine that reads your Answer Sheet will only see the solid black lines through the numbers beneath, not the written digits above. You must ensure that you code your answers or you will not receive any marks. There are further instructions and examples on the Answer Sheet.
5. The paper contains 20 questions. Five marks will be awarded for each correct answer. There is no penalty for giving an incorrect answer.
6. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the Senior Kangaroo should be sent to:

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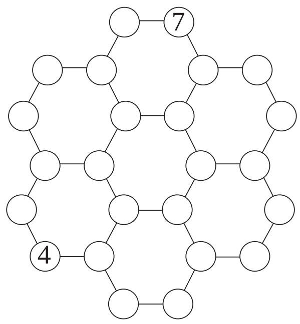
1. An integer is to be written in each circle of the network shown. The integers must be written so that the sum of the numbers at the end of each line segment is the same. Two of the integers have already been written. What is the total of all the integers in the completed diagram?



2. Three sportsmen called Primus, Secundus and Tertius take part in a race every day. Primus wears the number ‘1’ on his shirt, Secundus wears ‘2’ and Tertius wears ‘3’.

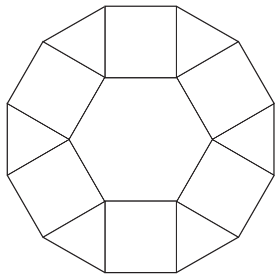
   On Saturday Primus wins, Secundus is second and Tertius is third. Using their shirt numbers this result is recorded as ‘123’.

   On Sunday Primus starts the race in the lead with Secundus in second. During Sunday's race Primus and Secundus change places exactly 9 times, Secundus and Tertius change places exactly 10 times while Primus and Tertius change places exactly 11 times.

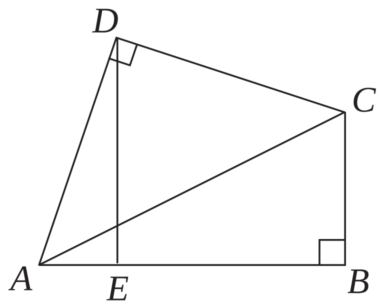
   How will Sunday's result be recorded?

3. All three-digit positive integers whose digit sum is 5 are listed in ascending order. What is the median of this list?

4. The figure shows a shape consisting of a regular hexagon of side 18 cm, six triangles and six squares. The outer perimeter of the shape is P cm. What is the value of P?



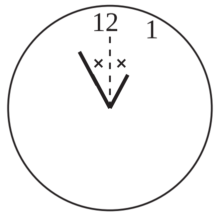
5. The figure shows a quadrilateral $ABCD$ in which $AD = DC$ and $\angle ADC = \angle ABC = 90°$. The point E is the foot of the perpendicular from D to AB. The length DE is 25. What is the area of quadrilateral $ABCD$?



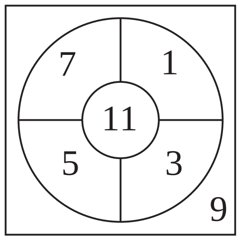
6. Winnie wrote all the integers from 1 to 2017 inclusive on a board. She then erased all the integers that are a multiple of 3. Next she reinstated all those integers that are a multiple of 6. Finally she erased all integers then on the board which are a multiple of 27. Of the 2017 integers that began in the list, how many are now missing?

7. Three rectangles are placed mutually adjacent and without gaps or overlaps to form a larger rectangle. One of the three rectangles has dimensions 70 by 110. Another of the rectangles has dimensions 40 by 80. What is the maximum perimeter of the third rectangle?

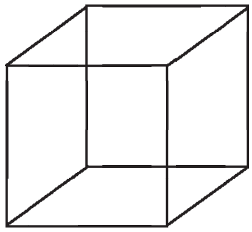
8. Priti is learning a new language called Tedio. During her one hour lesson, which started at midday, she looks at the clock and notices that the hour hand and the minute hand make exactly the same angle with the vertical, as shown in the diagram. How many whole seconds remain until the end of the lesson?



9. Robin shoots three arrows at a target. He earns points for each shot as shown in the figure. However, if any of his arrows miss the target or if any two of his arrows hit adjacent regions of the target, he scores a total of zero. How many different scores can he obtain?

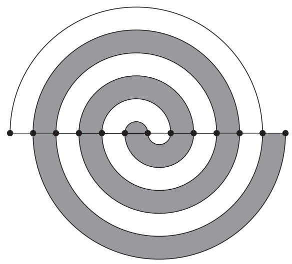


10. At each of the vertices of a cube sits a Bunchkin. Two Bunchkins are said to be adjacent if and only if they sit at either end of one of the cube's edges. Each Bunchkin is either a 'truther', who always tells the truth, or a 'liar', who always lies. All eight Bunchkins say 'I am adjacent to exactly two liars'. What is the maximum number of Bunchkins who are telling the truth?



11. An infinite arithmetic progression of positive integers contains the terms 7, 11, 15, 71, 75 and 79. The first term in the progression is 7. Kim writes down all the possible values of the one-hundredth term in the progression. What is the sum of the numbers Kim writes down?

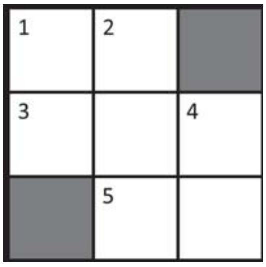
12. The pattern shown in the diagram is constructed using semicircles. Each semicircle has a diameter that lies on the horizontal axis shown and has one of the black dots at either end. The distance between each pair of adjacent black dots is 1 cm. The area, in cm^2, of the pattern that is shaded in grey is $\frac{1}{8}k\pi$. What is the value of k?



13. In the expression $\dfrac{k.a.n.g.a.r.o.o}{g.a.m.e}$, different letters stand for different non-zero digits but the same letter always stands for the same digit. What is the smallest possible integer value of the expression?

14. The set S is given by $S = \{1, 2, 3, 4, 5, 6\}$. A non-empty subset T of S has the property that it contains no pair of integers that share a common factor other than 1. How many distinct possibilities are there for T?

15. Each square in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are fulfilled. The digits used are not necessarily distinct.



ACROSS

1. A square
3. The answer to this Kangaroo question
5. A square

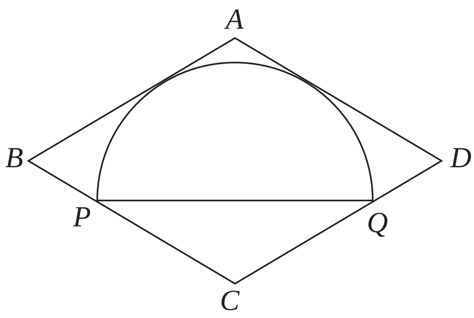
DOWN

1. 4 down minus eleven
2. One less than a cube
4. The highest common factor of 1 down and 4 down is greater than one

16. The curve $x^2 + y^2 = 25$ is drawn. Points on the curve whose x-coordinate and y-coordinate are both integers are marked with crosses. All of those crosses are joined in turn to create a convex polygon P. What is the area of P?

17. Matthew writes a list of all three-digit squares backwards. For example, in his list Matthew writes the three-digit square '625' as '526'. Norma looks at Matthew's list and notices that some of the numbers are prime numbers. What is the mean of those prime numbers in Matthew's list?

18. The diagram shows a semicircle with diameter PQ inscribed in a rhombus $ABCD$. The rhombus is tangent to the arc of the semicircle in two places. Points P and Q lie on sides BC and CD of the rhombus respectively. The line of symmetry of the semicircle is coincident with the diagonal AC of the rhombus. It is given that $\angle CBA = 60°$. The semicircle has radius 10. The area of the rhombus can be written in the form $a\sqrt{b}$ where a and b are integers and b is prime. What is the value of $ab + a + b$?



19. The sequence of functions $F_1(x)$, $F_2(x)$, … satisfies the following conditions:

$$F_1(x) = x, \qquad F_{n+1}(x) = \frac{1}{1 - F_n(x)}.$$

The integer C is a three-digit cube such that $F_C(C) = C$.
What is the largest possible value of C?

20. Let a, b and c be positive integers such that $a^2 = 2b^3 = 3c^5$. What is the minimum possible number of factors of abc (including 1 and abc)?